Practice for the GRE Math Subject Test!

2nd Edition

Charles Rambo

Preface

Thank you for checking out my practice exam! This was heavily influenced by the GR1268, GR0568, and GR8767 exams. I also used Rudin's *Principles of Mathematical Analysis*, Stewart's *Calculus*, *Counterexamples in Analysis* by Gelbaum and Olmsted, and *Counterexamples in Topology* by Steen and Seebach.

The answers can be found at

`rambotutoring.com/GREpracticeanswers.pdf`.

A hard copy of this practice test, another practice test, answers, and solutions is available on `Amazon.com` for \$10.75. The title is *Practice for the GRE Math Subject Test: Two Practice Tests and Solutions.*

To take care of a bit of shop work:

- Please email me at `charles.tutoring@gmail.com` if you find any mistakes.
- If you find this practice test helpful, please check out my GR1768 solutions at

  `rambotutoring.com/GR1768-solutions.pdf`.

  Also, you might be interested in my solutions booklet *GRE Mathematics Subject Test Solutions: Exams GR1268, GR0568, and GR9768.* It is on sale at `amazon.com`.
- For details about my tutoring business, check out my website `rambotutoring.com`. I tutor throughout North San Diego County.
- Please write a review for *Practice for the GRE Math Subject Test: Two Practice Tests and Solutions.* Feedback is extremely helpful!

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Escondido, California
April 2019

MATHEMATICS TEST

Time—170 minutes
66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is best and then completely fill in the corresponding space on the answer sheet.

Computation and scratch work may be done on a separate sheet of paper.

In this test:

(1) All logarithms with an unspecified base are natural logarithms, that is, with base e.

(2) The symbols $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.

---

1. $\displaystyle\lim_{n\to\infty}\left(n - n\cos\frac{1}{\sqrt{n}}\right)\sin\frac{1}{\sqrt{n}} =$

(A) $-\infty$ (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) ∞

---

2. Let $f(4)=3$ and $f'(4)=-2$. If $g(x)=\dfrac{xf(x^2)}{x^2+1}$, then $g'(2)=$

(A) $-\frac{89}{25}$

(B) $-\frac{13}{8}$

(C) $\frac{13}{8}$

(D) $\frac{75}{16}$

(E) $\frac{89}{25}$

---

3. Which of the following is an equation of a tangent line of $f(x)=x^3-3x^2+4x-3$ at an inflection point of f ?

(A) $y=x-2$

(B) $y=13x-8$

(C) $y=-x-2$

(D) $y=-13x+4$

(E) $y=13x+2$

---

4. $\displaystyle\int_e^{e^2}\log x\,dx =$

(A) $-e^2$ (B) $-e$ (C) 1 (D) e (E) e^2

5. Suppose $y = f(x)$ and $\dfrac{dy}{dx} = x - \dfrac{y}{2}$. If $f(1) = 2$, then $\lim_{x\to 1} \dfrac{f(x) - 2}{(x-1)^2} =$

(A) -1 (B) $-\dfrac{1}{2}$ (C) 0 (D) $\dfrac{1}{2}$ (E) 1

---

6. The volume of the parallelepiped which has edges parallel to and the same lengths as the position vectors $\mathbf{u} = \langle 0, 2, 0\rangle$, $\mathbf{v} = \langle 1, -1, 0\rangle$, and $\mathbf{w} = \langle -2, 2, 1\rangle$ is

(A) $\dfrac{1}{2}$ (B) $\dfrac{3}{4}$ (C) 2 (D) 15 (E) $13\sqrt{6}$

---

7. Consider the system

$$\begin{cases} y &= 2 \\ y &= a(x-b)^2 + c, \end{cases}$$

where a, b, and c are real numbers. For which of the following values of a, b, and c is there a solution to the system of equations?

(A) $a = -9$, $b = -4$, and $c = -5$

(B) $a = 7$, $b = -10$, and $c = 6$

(C) $a = 1$, $b = -6$, and $c = -4$

(D) $a = 2$, $b = 9$, and $c = 4$

(E) $a = -10$, $b = -10$, and $c = -6$

---

8. The lateral surface area of a cone is 6π and its slant height is 6. What is the radius of the cone's base?

(A) $\dfrac{1}{2}$ (B) 1 (C) $\dfrac{3}{2}$ (D) 2 (E) $\dfrac{12}{\pi}$

---

9. What is the measure of the angle between $\mathbf{u} = \langle 2, 0, 2\rangle$ and $\mathbf{v} = \langle 3\sqrt{2}, -6\sqrt{3}, 3\sqrt{2}\rangle$ in xyz-space?

(A) 0° (B) 30° (C) 45° (D) 60° (E) 90°

---

10. If U and V are 3-dimensional subspaces of $\mathbb{R}^5$, what are the possible dimensions of $U \cap V$?

(A) 0 (B) 1 (C) 0 or 1 (D) 1, 2, or 3 (E) 0, 1, 2, or 3

---

11. What is absolute minimum value of $f(x) = \dfrac{x^2 - 6x}{1 + |x+1|}$?

(A) -2 (B) -1 (B) $-\dfrac{1}{4}$ (C) 0 (D) $\dfrac{1}{4}$ (E) 7

12. Suppose T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}$ such that $T\begin{pmatrix} 1 \\ -3 \end{pmatrix} = 5$ and $T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = -2$. Then $T\begin{pmatrix} 0 \\ 2 \end{pmatrix} =$

(A) -4

(B) $-\dfrac{7}{2}$

(C) -1

(D) 3

(E) $\dfrac{9}{2}$

---

13. If $f(x) = (1-x)^{17}e^{2x}$, then $f^{(17)}(1) =$

(A) $-\left(17!2^{17}e^2\right)$

(B) $-\left(17!e^2\right)$

(C) 0

(D) $17!2^{17}e^2$

(E) $17!e^2$

---

14. A large class is to be divided into teams and each student must be a member of exactly one team. However, each student dislikes three of their classmates. Dislike between students need not be mutual. If the teams do not need to be equally sized, how many must be created so that no student is the teammate of someone they dislike?

(A) 4

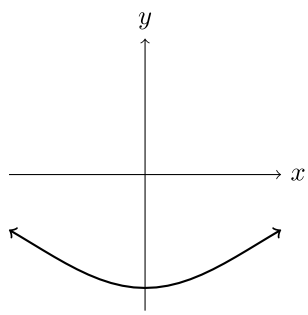
(B) 7

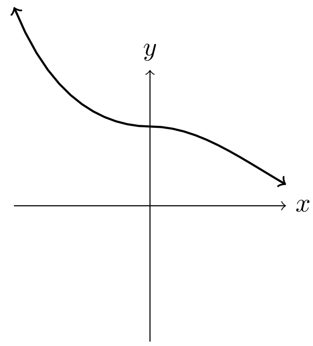
(C) 10

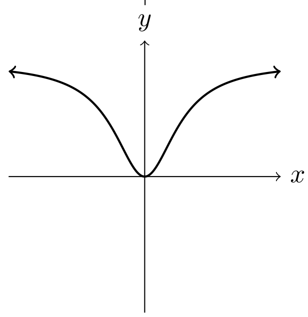
(D) 13

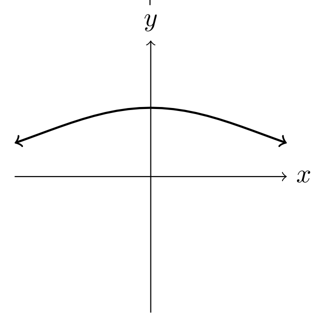
(E) 16

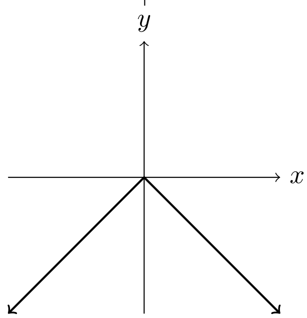
15. Which of the following could be the graph of a solution of $\dfrac{dy}{dx} = x\,|y|$?

(A) 

(B) 

(C) 

(D) 

(E) 

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16. Suppose $n\mathbb{Z} = \{0, n, -n, 2n, -2n, \ldots\}$ and $I_n = n\mathbb{Z} \cap [1, 1000]$. How many elements are contained in the set $I_6 \cup I_{15} \cup I_{25}$?

(A) 200 (B) 226 (C) 257 (D) 266 (E) 272

---

17. If $S = [0,1] \times [1,3]$, then $\displaystyle\iint_S xy^2\, dA =$

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{9}{2}$ (E) $\frac{13}{3}$

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18. Suppose $\mathcal{F}$ is the set of functions such that $f(x) = \dfrac{ax+b}{cx+d}$, where the coefficients $a,\ b,\ c$, and d are real numbers and $ad - bc = 1$. Which of the following are TRUE?

I. If f and g are in $\mathcal{F}$, then $f \circ g = g \circ f$.

II. There is a function i in $\mathcal{F}$ such that $i \circ f = f \circ i$ for all f in $\mathcal{F}$.

III. If f, g, and h are in $\mathcal{F}$, then $f \circ (g \circ h) = (f \circ g) \circ h$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

---

19. The region contained between the Lemniscate curve

$$r^2 = 4 \sin 2\theta$$

and the circle $r = \sqrt{2}$ has area

(A) $\sqrt{3} - \dfrac{\pi}{3}$

(B) $2\sqrt{3} - \dfrac{2\pi}{3}$

(C) $2\sqrt{3} - \dfrac{\pi\sqrt{2}}{3}$

(D) $2\sqrt{3} - \dfrac{\pi}{3}$

(E) $2 - \dfrac{\pi}{3}$

20. Let f be a differentiable real-valued function such that $f(3) = 7$ and $f'(x) \geq x$ for all positive x. What is the maximum possible value of $\int_0^3 f(x)\,dx$?

(A) 0

(B) $\frac{9}{2}$

(C) 12

(D) $\frac{25}{2}$

(E) 21

---

21. If $f : (0,1) \to (0,1]$, then which of the following could be TRUE?

I. f is one-to-one and onto.

II. The image of f is compact.

III. f is continuous, one-to-one, and onto.

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, and III

---

22. Suppose

$$f(x) = \begin{cases} \frac{2|x|}{5}, & \text{if } -1 \leq x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Then $\int_{-\infty}^{\infty} x f(x)\,dx =$

(A) $\frac{14}{15}$ (B) 1 (C) $\frac{6}{5}$ (D) $\frac{7}{3}$ (E) 3

---

23. For which value of n are there exactly two abelian groups of order n up to isomorphism?

(A) 4 (B) 7 (C) 8 (D) 12 (E) none of these

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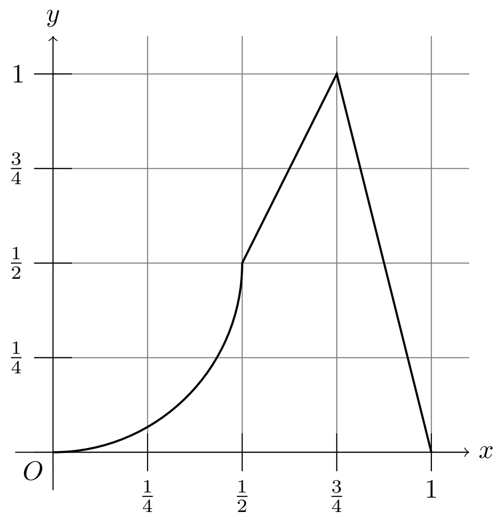


24. Above is the graph of $y = f(x)$. If $f(1+x) = f(x)$ for all real x, then $f'(25\pi) =$

(A) -16

(B) 0

(C) undefined

(D) 2

(E) not uniquely determined by the information given

25. The convergent sequence $\{x_n\}$ is defined by the recursive relationship $x_1 = 1$ and $x_{n+1} = \sqrt{15 - 2x_n}$ for all positive integers n. What is the value of $\lim\limits_{n\to\infty} x_n$?

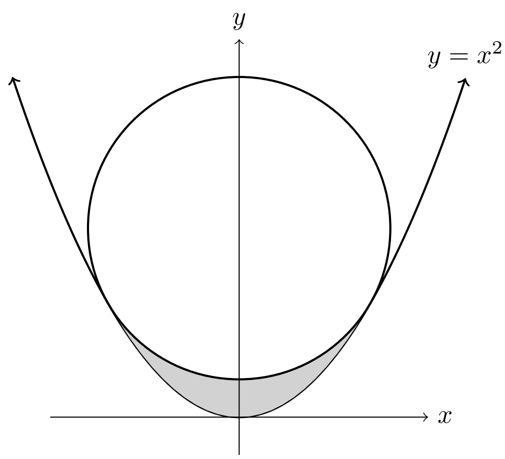
(A) -5 (B) -3 (C) 0 (D) 3 (E) 5

 GO ON TO THE NEXT PAGE.

x	$f(x)$	$f'(x)$	$g(x)$
-6	-5	1	-3
-2	$-\frac{5}{2}$	$\frac{1}{4}$	0
2	0	1	6
6	2	3	$\frac{13}{2}$

26. If f and g in the table above are inverses, then $(g' \circ g)(0) =$

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{3}$ (D) 1 (E) 4



27. A circle of radius 1 is tangent to $y = x^2$ at two points. What is the area of the region bounded by the parabola and circle?

(A) $\frac{2}{3} - \frac{\pi}{6}$ (B) $\sqrt{3} - \frac{11\pi}{24}$ (C) $\frac{3\sqrt{3}}{4} - \frac{\pi}{3}$ (D) $\frac{2}{3} - \frac{\pi}{12}$ (E) $\frac{11}{6} - \frac{\pi}{6}$

28. The arc length of the curve C from the point $(8, 1)$ to the point $(8e, e^2 - 8)$, where

$$C = \{(x, y) \in \mathbb{R}^2 : \ x = 8e^{t/2} \text{ and } y = e^t - 4t\} \ ,$$

is which of the following?

(A) $7 - e^2$ (B) $7 - e$ (C) $e^2 - 7$ (D) $e + 7$ (E) $e^2 + 7$

 GO ON TO THE NEXT PAGE.

29. Consider a segment of length 10. Points A and B are chosen randomly such that A and B divide the segment into three smaller segments. What is the probability that the three smaller segments could form the sides of a triangle?

(A) 0 (B) 10% (C) 25% (D) 50% (E) 80%

---

30. A discrete graph is complete if there is an edge connecting any pair of vertices. How many edges does a complete graph with 10 vertices have?

(A) 10 (B) 20 (C) 25 (D) 45 (E) 90

---

31. Suppose P is the set of polynomials with coefficients in $\mathbb{Z}_5$ and degree less than or equal to 7. If the operator D sends $p(x)$ in P to its derivative $p'(x)$, what are the dimensions of the null space n and range r of D ?

(A) $n = 1$ and $r = 6$

(B) $n = 1$ and $r = 7$

(C) $n = 2$ and $r = 5$

(D) $n = 2$ and $r = 6$

(E) $n = 3$ and $r = 5$

---

32. Consider the following algorithm, which takes two positive input integers `a` and `b` and prints a positive output integer.

```
input(a)
input(b)
   begin
      if a > b
         set max = a
         set min = b
      else
         set max = b
         set min = a
      while min > 0
         begin
            set r = max mod min
            replace max = min
            replace min = r
         end
      print a*b/max
   end
```

If `a = 20` and `b = 28` are the inputs of the following algorithm, what is the result?

(A) `4` (B) `5` (C) `7` (D) `140` (E) `560`

33. Let $\varphi(k)$ be a proposition which is either true or false depending on the integer k . Suppose that if $\varphi(k)$ is false then so is $\varphi(k-1)$. If there is some k_0 such that $\varphi(k_0)$ is true, what is the strongest conclusion that can be drawn?

(A) $\varphi(k)$ is true for all k .

(B) $\varphi(k_0+1)$ is true.

(C) $\varphi(k_0-1)$ is ture.

(D) $\varphi(k)$ is true for $k \le k_0$.

(E) $\varphi(k)$ is true for $k \ge k_0$.

---

34. Define

$$f(x) = \begin{cases} \sin \dfrac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

Let $I = \{(x, f(x)) \in \mathbb{R}^2 : \ -1 \le x \le 1\}$. Which of the following are TRUE?

I. The set I is connected.

II. The set I is path connected.

III. The set I is compact.

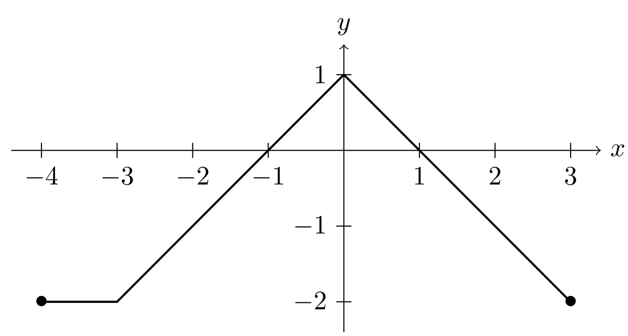
(A) I only

(B) III only

(C) I and II only

(D) I and III only

(E) I, II, and III



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35. The figure above shows the graph of the function f . Suppose that $g(x) = \displaystyle\int_0^x f(t)\ dt$. The absolute maximum of g is

(A) $g(-4)$ (B) $g(-3)$ (C) $g(-1)$ (D) $g(1)$ (E) $g(3)$

36. Let $f(x) = \int_0^{x^2} \sqrt{t}\sin\frac{1}{t}\, dt$, and let $I = [-1,0) \cup (0,1]$. Which of the following are TRUE?

I. f is bounded on the set I .

II. f' is bounded on the set I .

III. f'' is bounded on the set I .

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, and III

---

37. Suppose

$$A = \begin{pmatrix} 1 & 0 & -2 \\ c & -9 & -c \\ 0 & c & -1 \end{pmatrix} .$$

For what value(s) of c is A singular?

(A) -3 (B) -2 (C) -3 and 2 (D) -2 and 2 (E) -3 and 3

---

38. The region bounded by the non-negative x-axis and the function

$$f(x) = \frac{x}{1+x^3}$$

is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) π (D) 2π (E) ∞

39. Suppose

for all $\varepsilon > 0$ there exists a $\delta > 0$ such that for all x and y in D

$$|x - y| < \delta \quad \text{implies} \quad |f(x) - f(y)| < \varepsilon \ .$$

Consider the following statements.

A: For all $\varepsilon > 0$ there is a $\delta > 0$

B: For all x and y in D

C: $|x - y| < \delta$

D: $|f(x) - f(y)| \geq \varepsilon$

Using the letters listed above, the proposition originally stated is which of the following? Denote "not" by $\neg$.

(A) $A\Big(B(C \text{ or } D)\Big)$

(B) $A\Big(B(\neg C \text{ and } D)\Big)$

(C) $\neg A\Big(B(\neg C \text{ or } D)\Big)$

(D) $A\Big(\neg B(\neg C \text{ or } D)\Big)$

(E) $A\Big(B(\neg C \text{ or } \neg D)\Big)$

40. The radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(nx)^n}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot 2n}$ is

(A) 0 (B) $\dfrac{2}{e^2}$ (C) $\dfrac{2}{e}$ (D) $\dfrac{e}{2}$ (E) ∞

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41. Suppose V is a real vector space of finite dimension n. Call the set of matrices from V into itself $\mathcal{M}(V)$. Let T be in $\mathcal{M}(V)$. Consider the two subspaces

$$\mathcal{U} = \{X \in \mathcal{M}(V) : TX = XT\} \quad \text{and} \quad \mathcal{W} = \{TX - XT : X \in \mathcal{M}(V)\} .$$

Which of the following must be TRUE?

I. If V has a basis containing only eigenvectors of T then $\mathcal{U} = \mathcal{M}(V)$.

II. $\dim(\mathcal{U}) + \dim(\mathcal{W}) = n^2$

III. $\dim(\mathcal{U}) < n$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

---

42. If the finite group G contains a subgroup of order five but no element of G other than the identity is its own inverse, then the order of G could be

(A) 8 (B) 20 (C) 30 (D) 35 (E) 42

---

43. If $\zeta = e^{\frac{2\pi}{5}i}$, then $3 + 3\zeta + 12\zeta^2 + 12\zeta^3 + 12\zeta^4 + 9\zeta^5 + 5\zeta^6 =$

(A) $-4e^{\frac{2\pi i}{5}}$ (B) $-4e^{\frac{4\pi i}{5}}$ (C) 0 (D) $4e^{\frac{2\pi i}{5}}$ (E) $4e^{\frac{4\pi i}{5}}$

---

44. Suppose A is a 3×3 matrix such that

$$\det(A - \lambda I) = -\lambda^3 + 3\lambda^2 + \lambda - 3,$$

where I is the 3×3 identity matrix. Which of the following are TRUE of A ?

I. The trace of A is 3.

II. The determinate of A is -3.

III. The matrix A has eigenvalues -3 and 1.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

45. What is the general solution of

$$2\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 35y = 0 \ ?$$

(A) $y = C_1e^{-7x} + C_2e^{\frac{5}{2}x}$

(B) $y = C_1e^{-\frac{5}{2}x} + C_2e^{7x}$

(C) $y = C_1e^{-7x} + C_2e^{5x}$

(D) $y = C_1e^{-5x} + C_2e^{7x}$

(E) $y = C_1\cos(5x) + C_2\cos(7x)$

---

46. Let $f(x, y) = x^3 - y^3 + 3x^2y - x$ for all real x and y . Which of the following is TRUE of f ?

(A) There is an absolute minimum at $\left(\frac{1}{3}, \frac{1}{3}\right)$.

(B) There is a relative maximum at $\left(-\frac{1}{3}, -\frac{1}{3}\right)$.

(C) There is a saddle point at $\left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\right)$.

(D) There is an absolute maximum at $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$.

(E) All critical values are on the line $y = x$.

47. The difference $\log(1.1) - p(1.1)$, where

$$p(x) = x - 1 - \frac{1}{2}(x-1)^2 \ ,$$

is approximately

(A) $-\frac{1}{3} \times 10^{-3}$

(B) $-\frac{1}{4} \times 10^{-4}$

(C) $\frac{3}{8} \times 10^{-5}$

(D) $\frac{1}{4} \times 10^{-4}$

(E) $\frac{1}{3} \times 10^{-3}$

---

48. Suppose today is Wednesday. What day of the week will it be $10^{10^{10}}$ days from now?

(A) Sunday

(B) Monday

(C) Tuesday

(D) Wednesday

(E) Thursday

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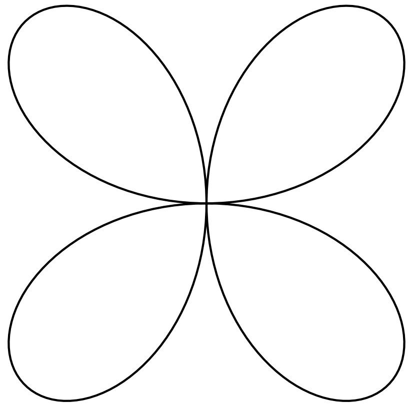
49. It takes Kate k days to write a GRE math practice test. It takes John j days to write a GRE math practice test. If Kate and John work on a practice test in alternating 2-day shifts, it takes them 10 days when Kate starts and 10.5 days when John starts. How long would it take the two to complete a practice test if Kate and John worked simultaneously?

(A) $\frac{9}{2}$ days (B) 5 days (C) $\frac{41}{8}$ days (D) $\frac{36}{7}$ days (E) 6 days

---

50. In the complex plane, let C be the circle $|z+2| = 3$ with positive (counterclockwise) orientation. Then $\displaystyle\int_C \frac{dz}{z^3(z-2)} =$

(A) $-\frac{\pi i}{4}$ (B) 0 (C) $\frac{3\pi i}{8}$ (D) $\frac{7\pi i}{8}$ (E) $2\pi i$



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51. A four-petaled rose curve has a group of symmetries which is isomorphic to the

(A) symmetric group S_4

(B) alternating group A_5

(C) cyclic group of order 4

(D) cyclic group of order 8

(E) dihedral group of 8 elements

---

52. Suppose the real-valued function f has a continuous derivative for all values of x in $\mathbb{R}$. Which of the following must be FALSE?

I. For some closed interval $[a, b]$ and every natural number N there exists an x in the interval $[a, b]$ such that $|f(x)| > N$.

II. For each real number c there are exactly two solutions of $f(x) = c$.

III. The limit $\lim_{x\to\infty} \dfrac{f(x)}{x} = \infty$ if and only if $\lim_{x\to\infty} f'(x) = \infty$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

---

53. Water drips out of a hole at the vertex of an upside down cone at a rate of 3 cm^3 per minute. The cone's height and radius are 2 cm and 1 cm, respectively. At what rate does the height of the water change when the water level is half a centimeter below the top of the cone? The volume of a cone is $V = \frac{\pi}{3} r^2 h$, where r is the radius and h is the height of the cone.

(A) $-\dfrac{48}{\pi}$ cm/min (B) $-\dfrac{4}{3\pi}$ cm/min (C) $-\dfrac{8}{3\pi}$ cm/min (D) $-\dfrac{24}{\pi}$ cm/min (E) $-\dfrac{16}{3\pi}$ cm/min

54. Suppose f is an analytic function of the complex variable $z = x + iy$ where x and y are real variables. If

$$f(z) = g(x, y) + e^y i \sin x$$

and $g(x, y)$ is a real-valued function of x and y, what is the value of

$$g\left(\frac{\pi}{2}, 7\right) - g(0, 0) \ ?$$

(A) $1 + e^7$

(B) $1 - e^7$

(C) 1

(D) $e^7 - 1$

(E) $2 - 2e^7$

---

55. Suppose A and B are $n \times n$ matrices with real entries. Which of the follow are TRUE?

I. The trace of A^2 is nonnegative.

II. If $A^2 = A$, then the trace of A is nonnegative.

III. The trace of AB is the product of the traces of A and B .

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

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56. Consider the independent random variables X_i such that either $X_i = 0$ or $X_i = 1$ and each event is equally as likely. Let

$$X = X_1 + X_2 + \ldots + X_{100} \ .$$

Which of the following values is largest?

(A) $\mathrm{Var}(X)$

(B) $100P\left(|X - 50| > 25\right)$

(C) $\displaystyle\sum_{k=0}^{100} k \binom{100}{k} \left(\frac{1}{2}\right)^k$

(D) $100P\left(X \geq 60\right)$

(E) 30

---

57. $\displaystyle\lim_{n\to\infty} \frac{1}{n} + \frac{1}{2+n} + \frac{1}{4+n} + \ldots + \frac{1}{3n} =$

(A) $\dfrac{1}{2}\log 2$ (B) $\dfrac{3}{4}$ (C) $\log\sqrt{3}$ (D) 1 (E) 2

58. Suppose A and A_k are subsets of $\mathbb{R}$ where k is any positive integer. Which of the following must be TRUE?

I. If A is closed, then A is compact.

II. If for each sequence $\{a_k\}$ with terms in A there is a strictly increasing function $\alpha : \mathbb{Z} \to \mathbb{Z}$ such that $\lim_{k\to\infty} a_{\alpha(k)}$ is in A, then A is compact.

III. If $B = \bigcup_{k=1}^{\infty} A_k$, then $\overline{B} = \bigcup_{k=1}^{\infty} \overline{A_k}$.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

---

59. The probability that a point (x, y) in $\mathbb{R}^2$ is chosen follows a uniform random distribution within the region described by the inequality $0 < |x| + |y| < 1$. What is the probability that $2(x + y) > 1$?

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{\sqrt{2}}{4}$ (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{3}{4}$

---

60. Let $\mathbf{F} = \left\langle y, -x, \frac{3}{\pi} \right\rangle$ be a vector field in xyz-space. What is the work done by $\mathbf{F}$ on a particle that moves along the path described by $\mathbf{r}(t) = \langle \cos t, \sin t, t^2 \rangle$ where t goes from 0 to $\frac{\pi}{2}$?

(A) $-\frac{\pi}{2}$ (B) $-\frac{\pi}{4}$ (C) 0 (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{2}$

---

61. There are 25 suitcases, 5 of which are damaged. Three suitcases are selected at random. What is the probability that exactly 2 are damaged?

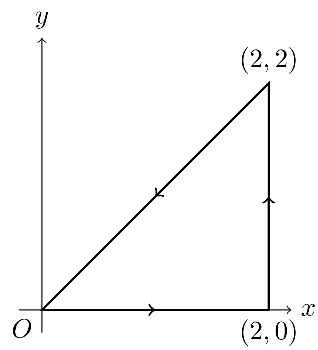
(A) $\frac{2}{69}$

(B) $\frac{1}{30}$

(C) $\frac{2}{23}$

(D) $\frac{12}{125}$

(E) $\frac{3}{25}$



62. Let C be the positively oriented path shown above. Then $\oint_C x\sin(x^2)\ dx + (3e^{y^2} - 2x)\ dy =$

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

63. The point on the plane $3x - 2y + z = 4$ closest to the origin is

(A) $(1, 2, 5)$

(B) $\left(\frac{6}{7}, -\frac{2}{7}, \frac{6}{7}\right)$

(C) $\left(\frac{5}{6}, \frac{4}{3}, \frac{25}{6}\right)$

(D) $(1, -3, -5)$

(E) $\left(\frac{6}{7}, -\frac{4}{7}, \frac{2}{7}\right)$

64. For each positive integer n, let f_n be the function defined on the interval $[0, 1]$ by $f_n(x) = \dfrac{nx}{1 + nx^2}$. Which of the following statements are TRUE?

I. The sequence $\{f_n\}$ converges point-wise on $[0, 1]$ to a limit function f.

II. The sequence $\{f_n\}$ converges uniformly on $[0, 1]$ to a limit function f.

III. $\left| \int_0^1 f_n(x)\ dx - \int_{\frac{1}{n}}^1 \lim_{k\to\infty} f_k(x)\ dx \right| \to 0$ as $n \to \infty$.

(A) I only

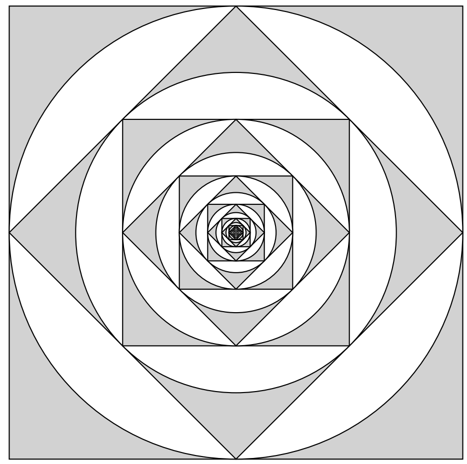
(B) III only

(C) I and II only

(D) I and III only

(E) I, II, and III

 GO ON TO THE NEXT PAGE.



65. The pattern in the figure above continues infinitely into the page. If the outer most square has sides of length 1, what is the total gray area of the figure?

(A) $1 - \dfrac{\pi}{4}$

(B) $2 - \dfrac{\pi}{3}$

(C) $2 - \dfrac{\pi}{2}$

(D) $\dfrac{1}{2}$

(E) $\dfrac{1+\pi}{4}$

GO ON TO THE NEXT PAGE.

$\cdot$	1	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$
1	1	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$
$\mathbf{i}$	$\mathbf{i}$	-1	$\mathbf{k}$	$-\mathbf{j}$
$\mathbf{j}$	$\mathbf{j}$	$-\mathbf{k}$	-1	$\mathbf{i}$
$\mathbf{k}$	$\mathbf{k}$	$\mathbf{j}$	$-\mathbf{i}$	-1

---

66. Suppose multiplication between 1, $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are as defined above. Which of the following are rings?

I. $\{a + b\sqrt[3]{4} : \ a$ and b are rational$\}$

II. The set of functions $f : \mathbb{R} \to \mathbb{R}$ under the standard function addition and multiplication defined by composition

III. $\{a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k} : \ a, b, c,$ and d are real$\}$

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

STOP

If you finished before time is called, you may check your work on this test.